Int. J. Heat Mass Transfer. Vol. 5, pp. 735–749. Pergamon Press 1962. Printed in Great Britain.

SURFACE EVAPORATION BY FORCED CONVECTION

I. SIMULTANEOUS HEAT AND MASS TRANSFER

C. G. M. SLESSER and D. CLELAND
Dept. of Chemical Technology, Royal College of Science and Technology, Glasgow, C.1

(*Received* 1 *December* 1961)

Abstract—The evaporation of liquid from a moving belt of liquid impregnated synthetic fibre has been studied as a problem in simultaneous heat and mass transfer by forced convection in a system which comprises several identical elements.

The novel apparatus is described which provides means of studying a variety of parameters under steady-state conditions. Expressions for heat- and mass-transfer rates are given, the equivalence of j-factors is demonstrated, and the index of the Schmidt number is shown to be 0·576. All results were correlated statistically.

NOMENCLATURE

a,	tow section perimeter, ft;
b,	regression coefficient, statistical correlations;
c,	concentration, lb mol/ft^3;
c_p,	heat capacity, Btu/lb degF;
c_f,	heat capacity of fibre, Btu/lb degF;
d,	tow width, ft;
D,	diffusion coefficient, ft^2/s;
$\mathrm{d}w/\mathrm{d}\theta$,	rate of evaporation, lb/ft^2 min;
f,	indicating functional relationship;
F,	fibre flow rate, lb/s;
h,	heat-transfer coefficient, Btu/ft^2 h degF;
j_h,	heat-transfer j-factor, dimensionless;
j_d,	mass-transfer j-factor, dimensionless;
k,	thermal conductivity, Btu ft/ft^2 h degF;
k, k',	constants, dimensionless;
k_g,	mass-transfer coefficient, lb mols/ft^2 atm h;
k_c,	mass-transfer coefficient, lb mols/ft^2 (unit concn.) h;
L,	length of tow on cage, ft;
M,	molecular weight;
m, m', n, n',	exponents;
N,	diffusion rate, lb mols/ft^{2}h;
P,	partial pressure, atm or mmHg;
S,	fibre speed, ft/s;
T_f,	fibre surface temperature, °F;
t_a,	air temperature, °F;
u,	air velocity, ft/unit time;
x,	moisture content, lb moisture/lb dry fibre;
z,	diffusing length;
Re,	Reynolds number, dimensionless;
Sc,	Schmidt number, dimensionless;
Nu,	Nusselt number, dimensionless;
Pr,	Prandtl number, dimensionless;
α,	tow thickness, ft;
β,	weight of tow per foot length, lb/ ft;
ρ,	density, lb/ft^3;
μ,	viscosity, lb/ft s;
θ,	time, s;
π,	total pressure, atm;
σ,	surface tension, lb;
λ,	latent heat, Btu/lb.

Subscripts

a,	diffusing gas;
b,	non-diffusing gas (air);
b_m,	log mean of b.

EVEN theoretically justified correlations, which have themselves been derived from experiments upon single-element systems, do not always prove to be accurate upon the larger scale. Such an instance is the design of evaporators where rates for single tubes differ from those of identical tubes in bundles. Thus, valuable *j*-factor correlations [1–4] which are well substantiated theoretically [5–7] may not necessarily be accurate in a multi-elemental system. To test this, a multi-elemental apparatus was devised in which simultaneous heat and mass transfer took place under steady-state conditions.

APPARATUS

Figures 1 and 2 show the apparatus. A continuous tow (belt) of wetted multifilament fibre was fed on to a rotating cage, and formed into a number of discrete spirals, then drawn off. Hot air blown from within the cage, passed radially outwards through the spaces between the spirals of tow, causing evaporation. The system afforded:

(1) Ten to fourteen elements, for each spiral may be so regarded.
(2) Steady-state conditions, for, being a flow system, it was not time dependent.
(3) Simple measurement of transfer rates, for if evaporation was held to the constant-rate period of drying, surface renewal of moisture was not rate controlling, and rates were dictated by gas boundary layer alone.
(4) Change of gas Schmidt number according to the choice of impregnating liquid.
(5) A drying surface of virtually rectangular cross section, whose dimensions did not change along the spirals.

Drying cage

The drying cage assembly (Fig. 1) consisted of a 4·375-in dia. hollow shaft over one end of which was fixed the cage, which comprised two 16·25-in dia. stainless-steel plates linked by sixteen radially spaced 0·5-in stainless-steel rods, making an enclosure 16 × 18 in. Hot air led into the shaft escaped through suitably placed holes fitted with blading, and through a diffuser. The final air distribution along the length of the cage is shown in Fig. 3, which reveals only minor random fluctuations. The unit was housed in an insulated box.

Tow advancement

The continuous tow was taken from a supply drum and fed by a pair of powered rollers (A) (Fig. 2) into a padding bath. After due immersion time, the tow was withdrawn by a second pair of powered rollers (B) which mangled the fibre

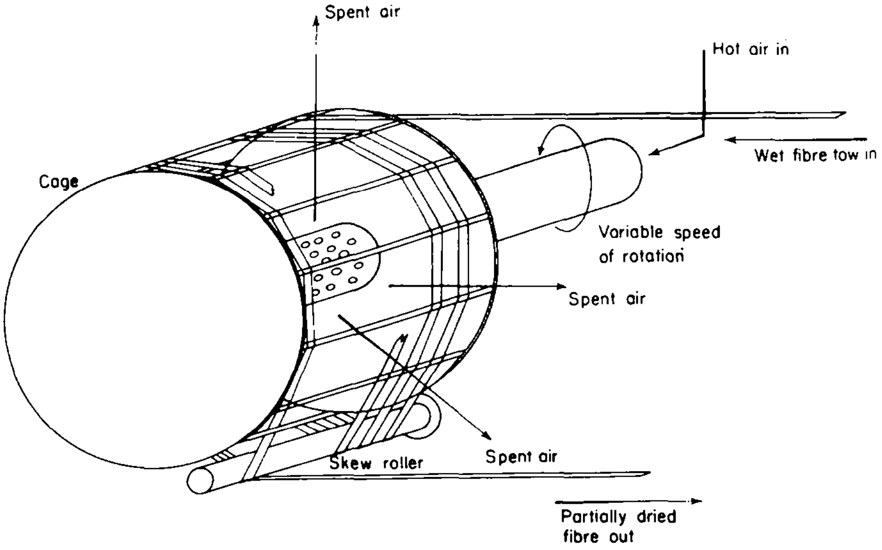


FIG. 1. Basic arrangement within drying compartment.

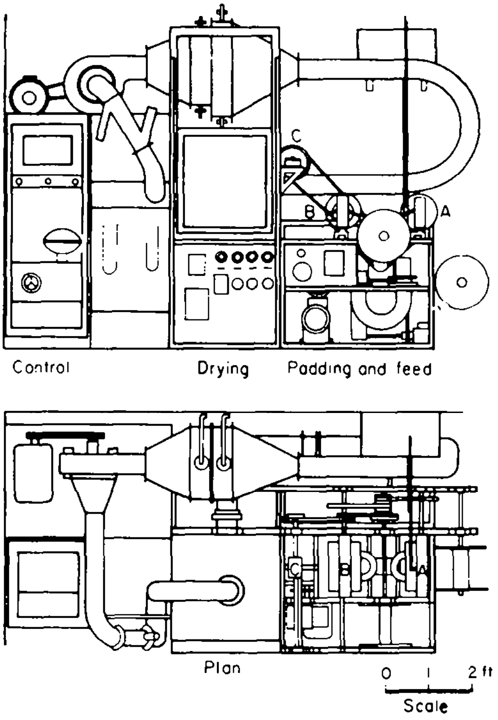


FIG. 2.

to remove surplus moisture, the squeeze being controlled to give the desired inlet moisture conditions to the drier. Before passing on to the cage, the tow passed over a tension roller, having the same peripheral speed as the cage, and 15 per cent greater than roller (B), which was itself adjustable to within ±5 per cent of the speed of (A). This permitted shrinkage and changes in padding time to be accommodated.

The tow was led round the cage, and once per spiral run over a skew roller, consisting of a 20 × 2 in dia. fluted stainless-steel shaft, free to rotate on an axis skew to the cage axis, but held rigid at one end by a ball bearing assembly. By adjustment of the skew, the number of spirals, and therefore the tow spacing, were readily altered.

From the last spiral the tow was led through spring-loaded off-take rollers to a collection drum.

Transmission

A ½ h.p. motor, geared down, and operated through an infinitely variable gear provided a wide range of cage speeds, and thus residence times. The system could be operated at peripheral speeds of from 9·9 to 80 ft/min, corresponding to a maximum input of 30 lb/h.

Air control

Steady-state air humidity was achieved by adjustment of a bleed manifold between drier and the 7-kW electric heater. For work with alcohols a steam-heated exchanger was used.

Padding system

The padding liquid was circulated from a constant-head tank to a bath of 1 ft^3 capacity in the form of a 4-in dia. glass U-tube with overflow ducts.

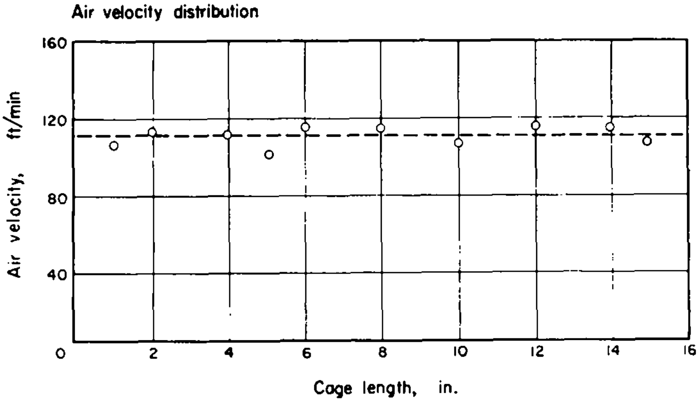


FIG. 3.

Instrumentation

Air temperature was controlled to within 0·5 degC by a three-term recorder–controller. The inlet-fibre temperature was taken as the padding-bath temperature, whose heat capacity was very large compared to the heat of wetting of the fibre. The outlet temperature was obtained by passing the tow over a plastic block (Fig. 4) in which was embedded the cold junction of a thermocouple, the hot junction being in the inlet air. The effect of heat by friction was tested and found negligible for the partially damp fibre.

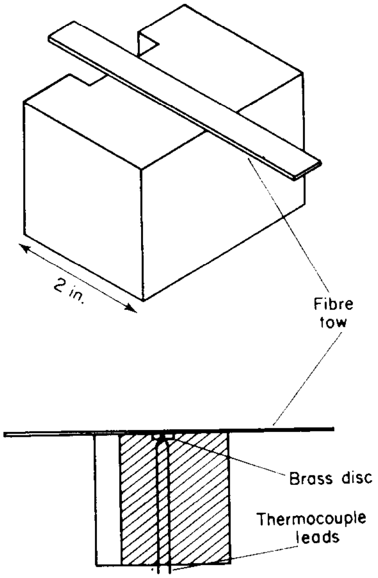


FIG. 4.

Humidity was measured with wet- and dry-bulb thermometers in the case of water-vapour experiments, and with a specially developed dew-point apparatus [8] for other vapours. Air flow was measured by a calibrated orifice placed in an 8-ft straight section of 6-in duct. Point values of air flow were measured by a hot-wire anemometer built after the method of Lowe and Hawes [9]. It was set up on the upper side of the cage, and was operated by remote control.

EXPERIMENTAL PROCEDURE

The tow was synthetic protein fibre (Ardil) which was available in 3000-ft lengths in various deniers from 3·5 to 22, and various filament numbers. Some comparative experiments were done with Terylene tow, but, since this material required an entirely different degree of stretch, it was abandoned.

The machine was operated continuously until absolutely steady-state conditions had been achieved for at least 30 min. Variation in air velocity, tow width, tow spacing, and partial pressure differences were studied for each liquid. The fibre was always maintained above the critical moisture content by suitable adjustment of the fibre flow rate through the machine.

This did not affect either heat- or mass-transfer rates. At the end of the steady-state run, the machine was shut down in an instant, and fibre samples swiftly taken from pre-determined points along the tow as it lay on the cage. Data from these samples gave the information to plot a rate-of-drying curve from which all other information could be derived. Moisture contents were determined by a gravimetric method and the results expressed on a dry basis.

THEORY

Heat transfer

As conditions of the experiment did not permit variations in gas Prandtl numbers, the value of the exponent for this number was chosen as 0·33 in common with other workers using the most nearly similar situations [10]. Thus heat transfer could be written:

$$Nu = k\,Re^{m}\,Pr^{0\cdot 33}. \qquad (1)$$

Heat-transfer coefficient

Consider Fig. 5 which depicts a short element of tow δL moving at a speed of S ft/s, so that it takes any point $\delta\theta$ seconds to move through the element. Let the fibre temperature as it enters the element be T_f and the air temperature t_a. Let the fibre enter at a moisture content of x, and lose δx in the element. Then, by heat balance:

$$a\,\delta L\,h\{t_a - [T_f + (\mathrm{d}T_f/\mathrm{d}L)\delta L]\}\,\delta\theta$$
$$= F\,\delta\theta[(\delta T_f\,c_f) + (x - \delta x)c_p\,\delta T_f + \delta x\lambda] \qquad (2)$$

cancelling $\delta\theta$ and second order terms, in the limit

$$a\,\mathrm{d}L\,h(t_a - T_f) = F(\mathrm{d}T_f\,c_f + \mathrm{d}x\,\lambda). \qquad (3)$$

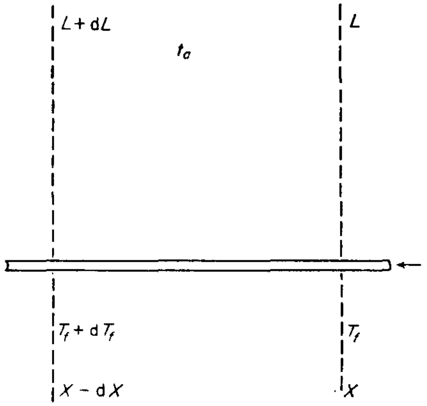


FIG. 5.

Now,

$$dL = S\,d\theta \text{ and } F = S\beta \tag{4}$$

Then,

$$d\theta = (\beta/F)\,dL, \tag{5}$$

i.e.

$$d\theta\,(a/\beta)\,h(t_a - T_f) = dT_f\,c_f + dx\,\lambda$$

and,

$$\frac{dx}{d\theta} = \frac{ah(t_a - T_f)}{\beta\lambda} - \frac{dT_f\,c_f}{d\theta\,\lambda}. \tag{6}$$

Now, at the constant rate, the surface temperature remains constant, hence

$$\frac{dT_f}{d\theta} = 0.$$

Thus, integrating,

$$h = \frac{F\Delta x\lambda}{aL(t_a - T_f)}. \tag{7}$$

Mass transfer

The general expression for diffusion, in terms of partial pressure is:

$$dP_a = \frac{RT}{D_{ab}\pi}(N_a P_b - N_b P_a)\,dz. \tag{8}$$

In this investigation the evaporating liquid (as vapour) was diffusing through a non-diffusing medium; hence $N_b = 0$, and N_a is constant (steady state). Then since $P_b = \pi - P_a$, and, if D_{ab} remains constant, it may be shown [11] that:

$$\frac{k_g M_{av} P_{b_m}}{u\rho_{av}} Sc^{1-n'} = j_d = k'\,Re^{m'-1}. \tag{9}$$

Mass-transfer coefficient

Consider again Fig. 5. Let the surface vapour pressure of the fibre be P_i at entry to the element and $P_i - (dP_i/dL)\delta L$ at the exit. Let P_a be the partial pressure of vapour in the ambient air-drying stream. Then, by mass balance:

$$k_g\left[\frac{(P_i + P_i - [dP_i/dL]\,\delta L}{2} - P_a\right] a\,\delta L\,\delta\theta\,M = F\delta\theta\delta x. \tag{10}$$

In the limit as δL becomes vanishingly small this becomes,

$$k_g(P_i - P_a)(a\,dL)M = F\,dx \tag{11}$$

and substituting from (5),

$$k_g(P_i - P_a)(a/\beta)M = dx/d\theta. \tag{12}$$

EXPERIMENTAL RESULTS

Both Gilliland [12] and Powell [4] have shown that when transfer is gas "film" controlled, the surface material does not affect transfer rates. This was corroborated using such dissimilar moisture-retaining materials as Ardil (100 per cent moisture) and Terylene (2–3 per cent moisture) (see Table 1).

Table 1

Temperature (degF)	194	167	122	95
Ardil $dw/d\theta$ [(lb/ft^2 min) × 10^3]	16·80	10·40	6·44	3·85
Terylene $dw/d\theta$ [(lb/ft^2 min) × 10^3]	16·68	10·78	6·67	3·59

Table 2 gives results of tests of experimental reliability demonstrating ±4 per cent.

Table 2

Air Temperature 167°F		Air Temperature 122°F	
dw/dθ [(ln/ft^2 min) $\times$ 10^3]	Deviation (%)	dw/dθ [(lb/ft^2 min) $\times$ 10^3]	Deviation (%)
10·36	+0·25	3·81	+1·0
10·62	+2·73	3·64	−3·7
10·04	−2·97	3·07	+2·7
10·35	+0·13	3·71	−1·7

Interaction of parameters

Each variable was tested in turn with all other factors constant. The exact relationship between rate of drying, expressed as pounds moisture evaporated per pound dry fibre, dx/dθ, and the variable was calculated statistically using the method of least squares as outlined by Davies [13]. The regression coefficient was calculated for the F-value, and compared with standard F-value statistical tables.

All preliminary experiments were done with the water–air system. Thereafter a range of lower alcohols and trichloroethylene were used to give a range of Schmidt numbers. The basis of choice was volatility, Schmidt number, and predictability of toxic and explosion hazards.

MASS TRANSFER

Correlation of air–water experiments

A series of runs (Table 3 and Fig. 6) was done at velocities from 37 to 140 ft/min at 167°F and constant partial-pressure difference, and gave the correlation

$$\mathrm{d}w/\mathrm{d}\theta = f(u)^b \qquad (13)$$

where b is the regression coefficient, and is shown to be highly significant. The influence of partial-pressure driving force on rates, and the possibility of an interaction between velocity and partial pressure were checked. The regression coefficient b was found to vary with partial pressure, though not in any systematic manner. To check whether there was any interaction between air velocity and ΔP_a, the highest and lowest values of b (0·783 at 29·4 mm, and 0·702 at 8·33 mm) were compared by the t-test, and shown to be without significance, so that the rate equation may be written

$$\mathrm{d}w/\mathrm{d}\theta = f(u^b \Delta P_a).$$

Table 3. *Correlation of air–water system*

Run no.	dw/dθ [(lb/ft^{2}min) $\times$ 10]	u (ft/min)	d (ft $\times$ 10^2)	t_a (degF)	h (Btu/ft^2 h degF)
1	13·9	137·0	2·09	167	9·23
2	10·05	114·0	2·09	167	8·95
3	9·80	88·3	2·09	167	7·41
4	7·66	74·7	2·09	167	6·80
5	6·22	54·0	2·09	167	5·30
6	11·02	137·0	2·63	167	8·30
7	10·38	114·0	2·63	167	7·83
8	9·29	88·3	2·63	167	7·0
9	7·34	73·6	2·63	167	5·54
10	5·80	54·3	2·63	167	4·37
11	12·2	137·0	3·93	167	8·40
12	9·53	114·0	3·93	167	6·78
13	8·15	88·3	3·93	167	5·80
14	6·36	74·0	3·93	167	5·10
15	5·34	54·3	3·93	167	4·10

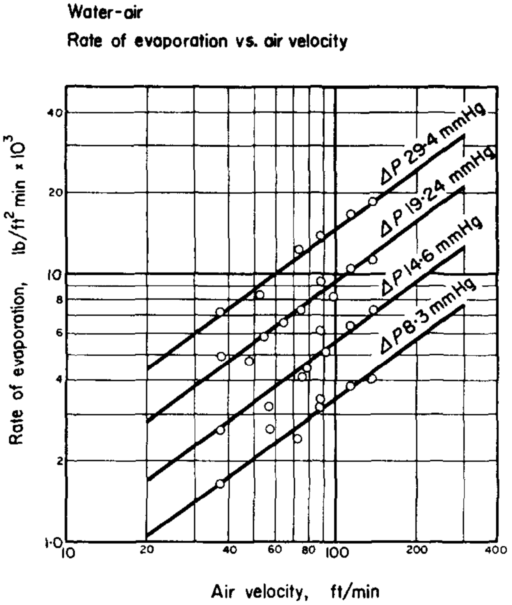


FIG. 6.

Table 4. Air–water system—influence of tow width

Run no.	d*w*/dθ [(lb/ft²min × 10³)]	u (ft/min)	$P_i - P_a$ (mm Hg)	d (ft × 10²)	t_a (degF)
1	3·17	88·0	8·3	2·63	95
2	2·45	74·4	8·3	2·63	95
3	2·62	57·1	8·3	2·63	95
4	1·66	39·4	8·3	2·63	95
5	4·03	138·0	8·3	2·63	95
6	3·82	114·0	8·3	2·63	95
7	3·46	88·3	8·3	2·63	95
8	5·01	93·0	14·6	2·63	122
9	4·46	68·4	14·6	2·63	122
10	2·64	36·6	14·6	2·63	122
11	4·11	75·7	14·6	2·63	122
12	6·11	89·0	14·6	2·63	122
13	6·42	115·0	14·6	2·63	122
14	7·33	137·0	14·6	2·63	122
15	9·29	88·3	19·24	2·63	167
16	10·38	114·0	19·24	2·63	167
17	11·02	137·0	19·24	2·63	167
18	8·05	98·7	19·24	2·63	167
19	7·34	73·6	19·24	2·63	167
20	6·58	64·0	19·24	2·63	167
21	5·80	54·3	19·24	2·63	167
22	4·93	37·0	19·24	2·63	167
23	4·75	48·0	19·24	2·63	167
24	18·43	137·0	29·4	2·63	194
25	16·85	114·0	29·4	2·63	194
26	13·75	88·7	29·4	2·63	194
27	12·10	72·0	29·4	2·63	194
28	8·14	64·0	29·4	2·63	194
29	7·19	37·0	29·4	2·63	194

The mean value of b was calculated by the method of William [14] which takes into account the different number of points chosen to calculate the original gradients. This yields

$$\mathrm{d}w/\mathrm{d}\theta = f(u^{0\cdot741}\Delta P_a). \tag{14}$$

The influence of tow width was studied, and the results given in Table 4 and Fig. 7, showed a relation

$$d\,\mathrm{d}w/\mathrm{d}\theta = f(ud)^{0\cdot74}. \tag{15}$$

The influence of tow spacing at constant air velocity was investigated, and showed only slight random variations.

Mass-transfer equation

The final relationship may be expressed by

$$\mathrm{d}w/\mathrm{d}\theta = 5\cdot58 \times 10^{-6}\frac{u^{0\cdot74}\Delta P_a}{d^{0\cdot26}}. \tag{16}$$

Before rendering this in dimensionless form the characteristic linear dimension must be decided upon. While on the cage, the fibre tow assumes a thin rectangular section. Powell [4], who investigated flow past rectangular-sectioned bodies, showed that little difference occurred between up- and down-stream surfaces. This, together with the known fact [15] that the experimental drag coefficient is considerably greater than the theoretical one, supports the view that the flow pattern is similar to the flow pattern past a cylinder, where the characteristic dimension is diameter. It therefore seems logical to take the tow width as the characteristic dimension of the present system.

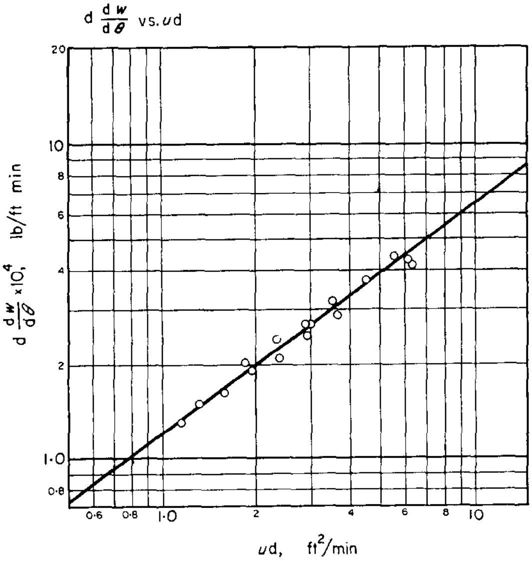


FIG. 7.

If the data is now rendered into dimensionless form (Table 5) it yields the expression,

$$\frac{k_g M_{av} P_{b_m}}{u \rho_{av}} = 0\cdot33\, Re^{-0\cdot26}. \tag{17}$$

Influence of Schmidt number

The work on the water–air system was repeated with other liquids and the results are given in Tables 6–10. The large volume of experimental data was used to confirm the exponent of the Reynolds number, and the regression coefficient b for each liquid is given in Table 11.

In forming a general expression for mass transfer, the variations in the power b were investigated and found to be statistically insignificant. Hence a weighted mean value of b was used and appropriate adjustments made to the constant in equation (16), as in Table 12.

In order to determine the effect of the Schmidt group, the mass-transfer data were plotted as in Fig. 8. The parameter of the parallel lines is the Schmidt group. The intercepts at a fixed value of the Reynolds number were statistically examined, and gave an exponent for the Schmidt group of 0·576. This result was tested and shown to be very highly significant. Thus the overall mass-transfer equation is shown to be:

$$j_d = \frac{k_g M_{av} P_{b_m}}{u \rho_{av}} Sc^{0\cdot576} = 0\cdot30\, Re^{-0\cdot28}. \tag{18}$$

HEAT TRANSFER

The heat-transfer data were readily correlated on the basis of equation (1). Table 13 summarizes the results obtained. The differences in the exponent m were investigated and found to be insignificant, and a weighted mean was calculated as before [14]. The results are shown in Table 14, and reveal slight variation in the coefficient k. The method of least squares was used to correlate the individual values and gave:

$$j_h = \frac{h}{c_p \rho_{av} u} Pr^{0\cdot67} = 0\cdot32\, Re^{-0\cdot30}. \tag{19}$$

Table 5. Air–water system—tow width data in dimensionless form

Run no.	h (Btu/ft^2 h degF)	k_g (moles/ft s atm × 10^4)	Re	Nu	Pr	$St_h \times 10^2$	$j_h \times 10^2$	$k_g M_{av} p_{b_m}/\rho u$ (× 10^2)	Sc	$j_d \times 10^2$
1	7·62	2·62	214	12·95	0·712	8·41	6·64	7·79	0·602	5·45
2	5·88	2·10	182	10·0	0·712	7·76	6·05	7·31	0·602	5·20
3	6·22	2·23	139	10·0	0·712	10·75	8·84	9·92	0·602	7·07
4	3·93	1·04	96	6·68	0·712	9·83	7·75	9·10	0·602	6·48
5	9·56	3·43	336	16·40	0·712	6·86	5·41	6·43	0·602	4·51
6	9·06	3·24	277	15·40	0·712	7·82	6·17	7·24	0·602	5·15
7	8·20	2·93	215	13·92	0·712	9·10	7·17	8·60	0·602	6·12
8	6·94	2·42	206	11·30	0·709	7·84	6·16	7·36	0·606	5·26
9	6·18	2·16	151	10·08	0·709	9·41	7·40	8·44	0·606	6·04
10	3·65	1·28	81	5·95	0·709	10·25	8·05	9·31	0·606	6·66
11	5·69	1·98	169	9·28	0·709	7·74	6·08	7·00	0·606	5·00
12	8·45	2·95	217	13·80	0·709	8·97	7·05	8·84	0·606	6·22
13	8·89	3·10	279	14·50	0·709	7·32	5·76	7·15	0·606	5·11
14	10·55	3·53	333	16·55	0·709	7·00	5·51	6·84	0·606	4·83
15	7·0	3·39	215	10·76	0·703	7·12	5·61	10·70	0·608	7·60
16	7·83	3·95	277	12·05	0·703	6·18	4·87	9·26	0·608	6·58
17	8·30	4·03	334	12·80	0·703	5·44	4·28	8·18	0·608	5·81
18	6·08	2·94	187	9·35	0·703	7·13	5·62	8·31	0·608	5·90
19	5·54	2·69	140	8·51	0·703	8·60	6·78	10·00	0·608	7·10
20	4·96	2·41	121	7·62	0·703	8·96	7·07	10·30	0·608	7·31
21	4·37	2·12	103	6·71	0·703	9·33	7·35	11·05	0·608	7·85
22	3·72	1·80	69	5·72	0·703	11·68	9·20	13·28	0·608	9·44
23	3·58	1·74	89	5·50	0·703	8·74	6·88	9·90	0·608	7·03
24	10·30	4·43	334	15·2	0·694	6·64	5·24	9·18	0·616	6·63
25	8·93	3·87	278	13·2	0·694	6·84	5·37	9·57	0·616	6·90
26	7·65	3·30	216	11·3	0·694	7·53	5·92	10·58	0·616	7·62
27	6·73	2·90	137	9·9	0·694	9·38	7·38	11·45	0·616	8·26
28	4·53	1·95	122	6·68	0·694	7·87	6·20	8·69	0·616	6·27
29	4·00	1·73	70	5·91	0·694	12·15	9·55	13·21	0·616	9·58

Table 6. Methanol–air system

Run no.	$dw/d\theta$ [(lb/ft^{2}min) × 10^3]	u (ft/min)	$P_i - P_a$ (mm Hg)	d (ft × 10^2)	t_a (degF)	h (Btu/ft^2 h degF)	k_g (moles/ ft^2 s atm × 10^4)	Re	Nu	Pr	St_h × 10^2	j_h × 10^2	$k_g M_{av} p_{b_m}/\rho u$	Sc	j_d × 10^2
1	3·31	137·0	44	2·63	167	9·9	2·98	334	15·40	0·694	6·63	5·15	5·78	1·01	5·78
2	2·78	114·0	44	2·63	167	8·52	2·60	277	13·25	0·694	6·90	5·36	6·09	1·01	6·09
3	2·45	88·3	44	2·63	167	7·24	2·21	215	11·25	0·694	7·55	5·86	6·67	1·01	6·67
4	2·09	72·0	44	2·63	167	6·17	1·88	176	9·60	0·694	7·86	6·10	6·96	1·01	6·96
5	4·75	137·0	67	2·63	194	10·58	2·80	334	16·10	0·703	6·85	5·36	5·54	0·98	5·45
6	4·11	114·0	67	2·63	194	9·13	2·42	277	13·90	0·703	7·12	5·58	5·76	0·98	5·66
7	3·46	88·3	67	2·63	194	7·69	2·04	215	11·72	0·703	7·77	6·03	6·28	0·98	6·16
8	2·95	72·0	67	2·63	194	6·57	1·74	176	10·05	0·703	8·10	6·34	6·58	0·98	6·47
9	2·45	137·0	32	2·63	122	10·60	3·02	334	16·30	0·709	6·90	5·40	6·01	0·97	5·90
10	2·16	114·0	32	2·63	122	9·45	2·66	277	14·50	0·709	7·39	5·78	6·38	0·97	6·27
11	1·87	88·3	32	2·63	122	8·20	2·32	215	12·60	0·709	8·29	6·48	7·20	0·97	7·07
12	1·48	72·0	32	2·63	122	6·47	1·82	176	9·95	0·709	7·96	6·24	6·88	0·97	6·76
13	1·66	137·0	23	2·63	95	10·40	2·86	334	16·00	0·710	6·75	5·28	5·83	0·965	5·70
14	1·35	114·0	23	2·63	95	8·45	2·32	277	13·00	0·710	6·60	5·16	5·71	0·965	5·58
15	1·27	88·3	23	2·63	95	7·90	2·18	215	12·15	0·710	7·94	6·21	6·92	0·965	6·77
16	1·02	72·0	23	2·63	95	6·36	1·75	176	9·80	0·710	7·84	6·16	6·80	0·965	6·65

Table 7. Ethanol–air system

Run no.	$dw/d\theta$ [(lb/ft^{2}min) × 10^3]	u (ft/min)	$P_i - P_a$ (mm Hg)	d (ft × 10^2)	t_a (degF)	h (Btu/ft^2 h degF)	k_g (moles/ ft^2 s atm × 10^4)	Re	Nu	Pr	St_h × 10^2	j_h × 10^2	$k_g M_{av} p_{b_m}/\rho u$	Sc	j_d × 10^2
1	4·11	137	44	2·63	194	10·2	2·59	336	15·7	0·694	6·70	5·21	4·91	1·34	5·98
2	2·78	88	44	2·63	194	7·16	1·81	214	11·0	0·694	7·39	5·75	5·37	1·34	6·48
3	2·38	70	44	2·63	194	5·91	1·49	172	9·1	0·694	7·65	5·95	5·54	1·34	6·74
4	2·16	57	44	2·63	194	5·36	1·36	132	8·25	0·694	8·98	6·98	6·08	1·34	7·40
5	3·03	137	37	2·63	167	10·70	2·24	336	6·96	0·703	6·96	5·50	4·42	1·32	5·25
6	2·45	88	37	2·63	167	8·65	1·83	214	8·80	0·703	8·80	6·95	5·67	1·32	6·83
7	2·23	70	37	2·63	167	7·90	1·67	172	10·05	0·703	10·05	7·95	6·46	1·32	7·80
8	1·73	57	37	2·63	167	6·12	1·30	132	10·10	0·703	10·10	7·98	6·08	1·32	7·33
9	2·59	137	28	2·63	122	11·80	2·56	336	7·97	0·708	7·97	6·30	5·29	1·31	6·24
10	1·95	88	28	2·63	122	8·80	1·92	214	9·00	0·708	9·00	7·10	5·81	1·31	6·96
11	1·73	70	28	2·63	122	7·80	1·73	172	9·91	0·708	9·91	7·86	6·57	1·31	7·88
12	1·51	57	28	2·63	122	6·85	1·50	132	10·10	0·708	10·10	7·98	6·88	1·31	8·35
13	2·09	137	22	2·63	95	12·10	2·63	336	7·80	0·710	7·80	6·18	5·35	1·30	6·40
14	1·59	88	22	2·63	95	9·20	1·99	214	9·26	0·710	9·26	7·35	6·34	1·30	7·57
15	1·30	70	22	2·63	95	7·50	1·63	172	9·24	0·710	9·40	7·45	6·50	1·30	7·76
16	1·15	57	22	2·63	95	6·65	1·45	132	10·90	0·710	10·90	8·56	6·97	1·30	8·35

Table 8. n-*Propanol–air system*

Run no.	$dw/d\theta$ [(lb/ft^{2}min) × 10^3]	u (ft/min)	$P_i - P_a$ (mm Hg)	d (ft × 10^2)	t_a (degF)	h (Btu/ft^2 h degF)	k_g (moles/ ft^2 s atm × 10^4)	*Re*	*Nu*	*Pr*	St_h × 10^2	j_h × 10^2	$k_g M_{av} p_{b_m}/\rho u$	*Sc*	j_d × 10^2
1	4·25	137	42·0	2·63	194	10·40	2·14	336	15·90	0·694	6·83	5·31	4·35	1·59	5·94
2	3·82	114	42·0	2·63	194	9·41	1·91	277	14·30	0·694	7·44	5·79	4·63	1·59	6·32
3	3·24	88	42·0	2·63	194	7·98	1·62	214	12·12	0·694	8·19	6·37	5·14	1·59	7·02
4	2·68	70	42·0	2·63	194	6·56	1·34	172	9·96	0·694	8·37	6·51	5·30	1·59	7·24
5	3·82	137	33·0	2·63	167	12·20	2·47	336	18·50	0·703	7·83	6·09	4·86	1·56	6·55
6	3·31	114	33·0	2·63	167	10·60	2·15	277	16·10	0·703	8·26	6·44	5·03	1·56	6·77
7	2·59	88	33·0	2·63	167	8·28	1·68	214	12·60	0·703	8·40	6·54	5·12	1·56	6·88
8	2·31	70	33·0	2·63	167	7·35	1·49	172	11·18	0·703	9·76	7·60	5·72	1·56	7·70
9	2·92	137	25·0	2·63	122	12·00	2·40	337	18·20	0·709	7·65	5·95	4·77	1·55	6·42
10	2·52	114	25·0	2·63	122	10·38	2·12	277	15·75	0·709	8·03	6·40	5·01	1·55	6·75
11	2·09	88	25·0	2·63	122	8·58	1·73	214	13·05	0·709	8·59	6·67	5·34	1·55	7·19
12	1·80	70	25·0	2·63	122	7·40	1·39	172	11·25	0·709	9·21	7·18	5·40	1·55	7·27
13	2·02	137	16·5	2·63	95	11·60	2·60	336	17·60	0·710	7·36	5·74	4·92	1·54	6·56
14	1·87	114	16·5	2·63	95	10·80	2·34	277	16·40	0·710	8·34	6·60	5·45	1·54	7·26
15	1·51	88	16·5	2·63	95	8·71	1·93	214	13·25	0·710	8·71	6·79	5·75	1·54	7·66
16	1·26	70	16·5	2·63	95	7·27	1·61	172	11·05	0·710	9·06	7·07	5·96	1·54	7·95

Table 9. n-*Butanol–air system*

Run no.	$dw/d\theta$ [(lb/ft^{2}min) × 10^3]	u (ft/min)	$P_i - P_a$ (mm Hg)	d (ft × 10^2)	t_a (degF)	h (Btu/ft^2 h degF)	k_g (moles/ ft^2 s atm × 10^4)	*Re*	*Nu*	*Pr*	St_h × 10^2	j_h × 10^2	$k_g M_{av} p_{b_m}/\rho u$	*Sc*	j_d × 10^2
1	4·83	137·0	40	2·63	194	11·40	2·045	336	17·3	0·694	7·41	5·77	4·19	1·77	6·11
2	4·17	114·0	40	2·63	194	9·70	1·78	277	14·9	0·694	7·73	6·02	4·36	1·77	6·35
3	3·10	88·3	40	2·63	194	7·30	1·32	214	11·10	0·694	7·45	5·80	5·08	1·77	7·41
4	2·45	55·0	40	2·63	194	6·10	1·105	132	9·25	0·694	10·50	8·17	5·60	1·77	8·17
5	3·46	137·0	28·5	2·63	167	10·90	2·075	336	16·7	0·703	7·05	5·55	4·07	1·74	5·95
6	3·02	114·0	28·5	2·63	167	9·56	1·815	277	14·6	0·703	7·41	5·84	4·28	1·74	6·18
7	2·45	88·3	28·3	2·63	167	8·42	1·595	214	12·9	0·703	8·54	6·73	4·90	1·74	7·07
8	1·84	55·0	28·5	2·63	167	5·82	1·105	132	8·9	0·703	9·58	7·55	5·28	1·74	7·54
9	1·08	137·0	9·0	2·63	95	10·70	2·05	336	16·4	0·712	6·86	5·45	3·88	1·72	5·85
10	0·97	114·0	9·0	2·63	95	9·65	1·84	277	14·8	0·712	7·51	5·96	4·24	1·72	6·07
11	0·79	88·3	9·0	2·63	95	7·86	1·50	214	12·1	0·712	7·96	6·32	4·35	1·72	6·39
12	0·54	55·0	9·0	2·63	95	5·36	1·14	132	8·2	0·712	8·76	6·95	5·36	1·72	7·70
13	1·73	137·0	14·5	2·63	122	10·40	2·04	336	15·9	0·709	6·86	5·43	4·05	1·73	5·97
14	1·48	114·0	14·5	2·63	122	8·90	1·75	277	13·6	0·709	6·94	5·50	4·20	1·73	6·03
15	1·15	88·3	14·5	2·63	122	6·95	1·36	214	10·65	0·709	7·02	5·56	4·22	1·73	6·07
16	0·86	55·0	14·5	2·63	122	5·65	1·11	132	8·65	0·709	9·30	7·36	5·48	1·73	7·88

Table 10. *Trichloroethylene–air system*

Run no.	$dw/d\theta$ [(lb/ft²min) × 10²]	u (ft/min)	$P_i - P_a$ (mm Hg)	d (ft × 10²)	t_a (degF)	h (Btu/ft² h degF)	k_g (moles/ft² s atm × 10⁴)	Re	Nu	Pr	St_h × 10²	j_h × 10²	$k_g M_{av} p_{b_m}/\rho u$	Sc	j_d × 10²
1	4·18	88·3	28·5	2·63	95	8·55	1·43	215	13·15	0·710	8·22	6·49	4·42	2·10	7·29
2	4·90	114·0	28·5	2·63	95	10·40	1·645	277	16·0	0·710	8·13	6·42	3·93	2·10	6·48
3	5·18	137·0	28·5	2·63	95	10·80	1·77	334	16·60	0·710	7·00	5·53	3·50	2·10	5·78
4	2·95	72·0	28·5	2·63	95	6·15	1·01	176	9·45	0·710	7·57	5·98	3·82	2·10	6·30
5	3·06	55·0	28·5	2·63	95	6·38	1·045	134	9·80	0·710	10·30	8·13	5·18	2·10	8·52
6	7·13	114·0	47·0	2·63	122	8·74	1·46	277	13·40	0·709	6·84	5·40	3·34	2·11	5·48
7	8·36	137·0	47·0	2·63	122	10·25	1·715	334	15·75	0·709	6·56	5·18	3·25	2·11	5·34
8	5·98	88·3	47·0	2·63	122	7·32	1·225	215	11·20	0·709	7·34	5·80	3·64	2·11	5·97
9	4·75	72·0	47·0	2·63	122	5·83	0·975	176	8·95	0·709	7·18	5·67	3·48	2·11	5·71
10	4·61	55·0	47·0	2·63	122	5·65	0·95	134	8·68	0·709	9·14	7·21	4·48	2·11	7·35
11	8·00	72·0	69·0	2·63	167	6·08	1·265	176	9·45	0·703	7·64	6·03	4·78	2·12	7·74
12	10·30	88·3	69·0	2·63	167	7·85	1·445	215	12·20	0·703	8·07	6·37	4·38	2·12	7·25
13	11·80	114·0	69·0	2·63	167	9·00	1·655	277	14·40	0·703	7·20	5·68	3·87	2·12	6·30
14	13·70	137·0	69·0	2·63	167	10·42	1·92	334	16·20	0·703	6·90	5·45	3·74	2·12	6·18
15	7·35	55·0	69·0	2·63	167	5·60	1·035	134	8·70	0·703	9·20	7·26	4·98	2·12	8·26
16	16·80	114·0	96·0	2·63	194	9·85	1·690	277	15·15	0·694	7·80	6·16	4·12	2·14	6·89
17	19·50	137·0	96·0	2·63	194	11·40	1·980	334	17·5	0·694	7·45	5·88	4·03	2·14	6·70
18	15·10	88·3	96·0	2·63	194	8·87	1·53	215	13·62	0·694	9·02	7·12	4·85	2·14	8·08
19	12·30	72·0	96·0	2·63	194	7·22	1·23	176	11·10	0·694	9·08	7·16	4·76	2·14	7·94
20	9·50	55·0	96·0	2·63	194	5·57	0·96	134	8·55	0·694	9·20	7·26	4·93	2·14	8·22

Table 11

Water		Methanol		Ethanol		n-Propanol		n-Butanol		Trichloroethane	
ΔP_a	b	ΔP_a	b	ΔP_a	b	ΔP_a	b	ΔP_a	b	ΔP_a	b
8·30	0·702	67	0·731	44	0·751	42	0·69	40	0·754	96	0·766
14·6	0·733	44	0·706	37	0·690	33	0·77	28·5	0·68	69	0·710
19·2	0·698	32	0·758	28	0·640	25	0·72	14·5	0·76	47	0·676
29·4	0·783	23	0·698	22	0·695	16·5	0·76	9·0	0·771	28·5	0·691
Weighted mean	0·741		0·723		0·694		0·735		0·741		0·711

ΔP_a is in mm Hg.

Table 12*

Liquid	constant × 10^6	b
Water	0·613	0·72
Methanol	0·856	0·72
Ethanol	1·08	0·72
Propanol	1·23	0·72
Butanol	1·32	0·72
Trichloroethylene	2·13	0·72

* From equation (16).

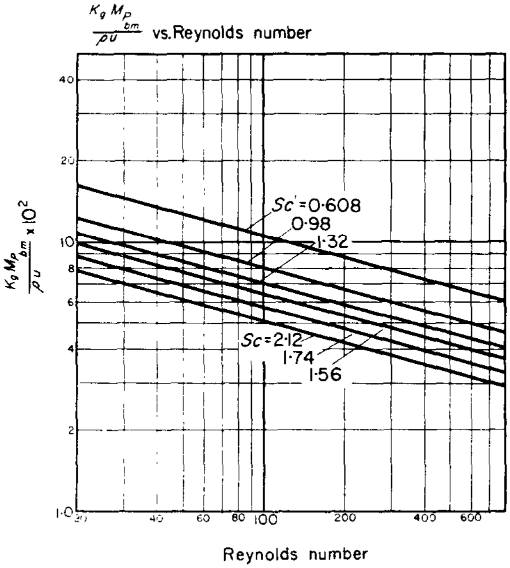


FIG. 8.

Table 13

k	m
0·34	0·68
0·25	0·73
0·28	0·68
0·32	0·71
0·38	0·67
0·30	0·71

Table 14

k	m
0·31	0·70
0·30	0·70
0·25	0·70
0·28	0·70
0·32	0·70
0·32	0·70

DISCUSSION

Equation (13) for the rate of evaporation of water may be compared with Powell's [4] result, also with rectangular shapes.

His exponent was 0·69, against 0·741 here, and, since his results were not statistically correlated, the divergence could easily come within range of experimental values.

Comparative results for heat- and mass-transfer correlations on related single-element studies are given in Tables 15 and 16. The most closely related mass-transfer study, that of Lohrisch [17] shows the same dependence on Schmidt number (0·56 as against 0·576 here), but indicates considerable divergence in the Reynolds number influence, though the range and Reynolds number studied was different in the two cases. Fig. 9 shows the results plotted as j_d versus Reynolds number, and it may be seen that Lohrisch's correlation (line 1) gives a line of the same slope as the present correlation (line 5). If the equation of line 5 [i.e. equation (18)] be divided by 2, lines 1 and 5 coincide. This factor of 2 as between transfer from a strip and transfer from a cylinder is not surprising, and the marked similarity of the plotted lines possibly justifies an extension of either expression to wider ranges of Reynolds number.

Table 15

System	k'	n'	m'	Ref.
Cylinder	0·28	−0·56	−0·40	[17]
Sphere	0·43	−0·67	−0·44	[19]
Rectangular-sectioned object	0·30	−0·567	−0·28	Present

Table 16

System	k	m	n	Ref.
Flow past rectangular sectioned objects	0·32	0·70	0·33	Present
Flow past cylinders	0·26	0·60	0·30	[20]
Flow through pipes	0·023	0·80	0·33	[21]

Constants as in equation (1).

The value of 0·576 for the exponent of the Schmidt number is of considerable interest when the experimental result has shown to be statistically so significant, and it supports

Gilliland and Sherwood [18] and Lohrisch [17].

The validity of the j-factor similarity was tested by summarizing all results in the form:

$$j_h = k\, Re^m \tag{20}$$

$$j_d = k'\, Re^{m'}. \tag{21}$$

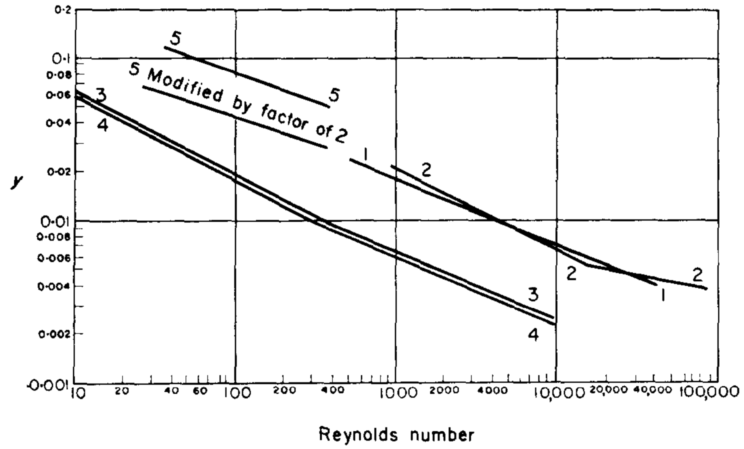
These values are shown in Table 18 (see below), and it can be shown statistically that the difference between the corresponding values of m and m' are insignificant when compared to the range of Reynolds numbers of the points used for the correlations. Hence mean values were taken for each liquid, and corresponding values of k and k' appropriately adjusted, as in Table 17, so that the ratio $k'/k = j_d/j_h$. It may be seen that the ratio at all times exceeds 1, except in the case of water, by between 2·5 and 8 per cent. The correlations for this multi-elemental system are thus not only closely similar to single-element systems, but offer extremely good and well-substantiated evidence of the equivalence of the

Table 17

Liquid	k	k'	$(j_d/j_h) = (k'/k)$
Water	0·304	0·300	0·99
Methanol	0·27	0·29	1·08
Ethanol	0·365	0·374	1·025
Propanol	0·277	0·300	1·08
Butanol	0·396	0·421	1·065
Trichloroethylene	0·318	0·342	1·08

Table 18

System water–air	j_d/j_h	Ref.
Through circulation drying of spheres	0·93	[22]
Evaporation from a porous block	0·91	[23]
Evaporation from flat surface	0·94	[24]
Present	0·99	Present



Legend

No.	System	Medium	y
1	Flow past cylinders	Gases	$J_d = \frac{K_g M_{P_{bm}}}{\rho u} Sc^{0 \cdot 56}$
2	Flow parallel to plates	Gases	$J_d = \frac{K_g M_{P_{bm}}}{\rho u}$
3	Flow through packed solids	Gases	$J_h (10^{-1}) = \frac{h}{C_p G} Pr^{3/2} (10^{-1})$
4	Flow through packed solids	Gases	$J_d (10^{-1}) = \frac{K}{G_m} Sc^{2/3} (10^{-1})$
5	Flow past rect. sectioned object	Gases	$J_d = \frac{K_g M_{P_{bm}}}{\rho u} Sc^{0 \cdot 567}$

FIG. 9.

j-factors. Table 18 gives some comparable results obtained for the water–air system by other workers. It is to be noted that this system is one of the few to be theoretically analysed, and was shown by Callaghan [21] to have a theoretical j_d/j_h ratio of 1·05.

SUMMARY

(1) The j-factor correlation has been applied to a multi-elemental system and shown to be valid.

(2) The influence of Schmidt number has been studied, and by means of rigorous statistical analysis the exponent has been shown to be 0·576, a value which when tested statistically was demonstrated to be very highly significant.

(3) Mass- and heat-transfer equations for this geometry have been evaluated, and it has been shown that j_d/j_h has a mean value such that:

$$j_d = 1{\cdot}05\, j_h = 0{\cdot}30\, Re^{-0{\cdot}28}.$$

ACKNOWLEDGEMENTS

The authors are grateful to Mr. David Smith, of the Department of Chemical Technology, R.C.S.T., for his generous and valuable help and advice in making statistical correlations, to Messrs. Imperial Chemical Industries Ltd. who aided the work both by a grant and by the gift of equipment and continuous Ardil and Terylene tow, to Messrs. J. & P. Coats, Paisley, who granted a fellowship to one of us, and to the workshop staff of the Chemical Technology Department who assisted in the construction of the machine.

REFERENCES

1. T. H. Chilton and A. P. Colburn, *Industr. Engng Chem.* **26**, 1183 (1934).
2. R. W. Powell, *Trans. Instn Chem. Engrs, Lond.* **13**, 175 (1935).
3. F. W. Dittus and L. M. K. Boelter, *Univ. Calif. Publ.* **2**, 443 (1952).
4. R. W. Powell, *Trans. Instn Chem. Engrs, Lond.* **18**, 36 (1940).
5. E. R. G. Eckert, *Introduction to the Transfer of Heat and Mass* pp. 249–251. McGraw-Hill, New York.
6. E. Schmidt, *Gesundheitsing*, **52**, 525–529 (1929).
7. W. Nusselt, *Z. Angew. Math. Mech.* **10**, 105–121 (1930).
9. C. G. M. Slesser and D. Cleland, *Chem. & Ind.* 551–552 (1962).
9. E. Lowe and J. R. Hawes, *Food. Tech.* **3**, 241–243 (1949).
10. D. Q. Kern, *Heat Transfer* p. 50. McGraw-Hill, New York (1950).
11. R. E. Treybal, *Mass Transfer Operations* p. 45. McGraw-Hill, New York (1955).
12. E. R. Gilliland, *Industr. Engng Chem.* **30**, 506 (1938).
13. O. L. Davies, *Statistical Methods in Research and Production* pp. 194–196. Oliver and Boyd, Edinburgh.
14. E. J. William, *Regression Analysis* p. 132. Wiley, New York.
15. I. M. Milne-Thompson, *Theoretical Hydrodynamics* pp. 299–301. McMillan, London.
16. J. N. Coulson and J. P. Richardson, *Chemical Engineering* Vol. 1, p. 239 Pergamon Press, Oxford (1961).
17. W. Lohrisch, *Mitt. Forsch.* **322**, 46 (1929).
18. E. R. Gilliland and T. K. Sherwood, *Industr. Engng Chem.* **26**, 516 (1934).
19. G. C. Williams, Thesis, M.I.T. (1942).
20. E. Griffiths and J. H. Awberry, *Proc. Instn Mech. Engrs, Lond.* **125**, 319–382 (1933).
21. E. E. Callighan, *N.A.C.A.* No. 3045 (1953).
22. E. W. Gamson, *et al.*, *Trans. Amer. Inst. Chem. Engrs*, **39**, 1 (1943).
23. P. M. Heertjees and W. P. Ringens, *Chem. Engng Sci.* **5**, 226 (1956).
24. Coles and Ruggeri, Unpublished report, Lewis Flight Laboratories, Ohio.

Zusammenfassung—Das Problem des gleichzeitigen Wärme- und Stoffüberganges bei Zwangskonvektion liess sich bei der Verdunstung einer Flüssigkeit von einem bewegten flüssigkeitsgetränkten Band aus synthetischen Fasern studieren. Die neuartige Anordnung ist beschrieben. Sie erlaubt die Untersuchung einer Vielzahl von Parametern unter stationären Bedingungen. Gleichungen für Wärme- und Stoffübergang sind angegeben, die Äquivalenz der j-Faktoren ist nachgewiesen, der Exponent der Schmidt-Zahl ergibt sich zu 0,576. Alle Ergebnisse wurden statistisch korreliert.

Аннотация—Испарение жидкости с поверхности движущейся ленты синтетического волокна, пропитанного жидкостью, изучалось как задача об одновременном тепло-и массообмене при вынужденной конвекции в системе, состоящей из нескольких идентичных элементов.

Описывается новый прибор, позволяющий находить различные параметры стационарного переноса тепла и вещества. Приводятся выражения для удельных потоков тепла и массы, показана эквивалентность коэффициентов j_h и j_d. Показатель степени числа Шмидта равен 0,576. Все результаты обработаны статистически.